Source depth and the spatial coherence of ambient noise in the ocean

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An analytical model is developed for the vertical coherence of the ambient noise field generated by a plane of sources at a finite depth beneath the ocean surface. To clarify the effects of source depth on the noise field, the relatively simple case of a semi-infinite ocean with an isovelocity profile is considered. The expression derived for the coherence is exact; it depends on the source depth explicitly, and it includes the homogeneous and inhomogeneous components of the field. When the sources are shallow, that is, the source depth is much less than a wavelength and the source–image pairs act as dipoles, the coherence is an oscillatory function of frequency, consistent with an earlier theory of noise coherence in deep water. With deeper sources, the dipole description fails and the coherence function becomes approximately independent of frequency. This change of character suggests that the spatial structure of the noise field at depth in the ocean could be inverted to yield information on the acoustic properties of the bubble sources associated with breaking surface waves.
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INTRODUCTION

Ambient noise in shallow water is strongly influenced by the proximity of the seabed, which modifies the spatial structure of the noise field in the vertical.[1,2] This effect has been exploited recently as the basis of a noise inversion procedure for quantitatively determining the phase speeds of the compressional and shear waves in the seabed.[3,4]

In addition to the bottom, an important factor affecting the spatial coherence of ambient noise in the ocean is the depth of the noise sources. Concentrating on wind-related sources, particularly bubble formation arising from breaking wave events,[5–7] it is likely that the sources are at depths between a few centimetres to a meter or so, depending on surface conditions. At frequencies such that the source depth is a small fraction of a wavelength, a source and its negative image in the surface act as a dipole[8–10] and exhibit the characteristic dipolar radiation pattern, showing a null in the horizontal; but at higher frequencies, where the source depth is a significant fraction of a wavelength or greater, the dipole description is no longer valid. The transition from dipole behavior to two monopoles of opposite sign introduces some interesting features into the vertical coherence structure of the ambient noise field.

The earliest theoretical model for the spatial coherence of surface-generated noise in the ocean was developed by Cron and Sherman.[11,12] They represented the ocean as a semi-infinite half-space with a uniform sound speed profile, and assumed that the noise sources were distributed at random points in a layer lying immediately beneath the surface. Their treatment is approximate in that it neglects the spatially inhomogeneous component in the noise field; and the depth of the source layer was taken to be so small that it simply cancels out of their expression for the noise coherence function. Similarly, in several of the more sophisticated noise models that were to follow, the source depth is absent from the theoretical description of the coherence.

The present article evolved out of our attempts to invert the vertical coherence of ambient noise in shallow water for the geoacoustic parameters of the bottom.[4] To perform reasonable inversions, especially at the higher frequencies (i.e., around 600 Hz and above with a hydrophone spacing of 1 m), we found that it is necessary to include explicitly the source depth in our noise model. Indeed, by so doing, the depth of the sources became one of our inversion parameters, along with those describing the geoacoustic properties of the bottom.

To illustrate the effects of source depth on the vertical spatial structure of the noise a relatively simple model is developed here, which is an extension of the original deep-water model of Cron and Sherman;[11,12] that is to say, to clarify the relationship between source depth and vertical coherence, the influence of the bottom has been removed from the problem entirely. The analysis leads to an exact, closed-form expression for the vertical coherence of the noise in which the source depth appears explicitly as a parameter. This result for the coherence function includes terms representing both the homogeneous and inhomogeneous components of the noise field, although the latter is entirely negligible at receiver depths greater than a wavelength. In the limit of zero source depth, the homogeneous term reduces to the form derived by Cron and Sherman,[11,12] as required. A finite source depth, on the other hand, introduces a significant departure from Cron and Sherman's result.

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I. PROPAGATION IN A SEMI-INFINITE ISOTROPIC OCEAN

A fundamental element of any ambient noise model is the Green's function representing the propagation between a single source and a receiver. In the case of interest here, in which a monopole source radiates sound beneath a pressure-release sea surface, the Green's function can immediately be stated as the difference between two spherical spreading terms, each of the form e^{jkR}/R, where k is the acoustic wave number, $j=\sqrt{-1}$, and R is the distance between receiver and source in one case or receiver and image in the other. This formulation of the Green's function was Cron and Sherman's[11,12] starting point, but, as they pointed out, it leads to unmanageable integrals in the analysis of the noise and forces the introduction of an approximation that we wish to avoid.

An alternative expression for the Green's function, G, of a single source is obtained by solving the Helmholtz equation using a cylindrical coordinate system with the origin at the surface and the axis vertical, passing through the receiver. Depth is positive downward. The details of the analysis are given in the Appendix. The solution for G is in the form of an inversion integral over horizontal wave number, p:

$$G=-\frac{Q}{4\pi j}\int_0^\infty \frac{p}{\eta}\{e^{j\eta(z-z')}-e^{j\eta(z+z')}\}J_0(pr)dp,\quad z>z', \tag{1}$$

where r is horizontal range, z and z' are receiver and source depths, respectively, $J_0(\cdots)$ is the Bessel function of the first kind of order zero, Q is the source strength, and the vertical wave number is

$$\eta=\sqrt{k^2-p^2},\quad \mathrm{Im}(\eta)>0. \tag{2}$$

The inequality in Eq. (2) ensures that the solution for G converges at infinite depth. In Eq. (1) it is apparent that G is a function of frequency, since the radical η depends on the wave number, $k=\omega/c$, where ω is angular frequency and c is sound speed in the medium. This frequency dependence arises because G is the Fourier transform (with respect to time) of the acoustic pulse arrival at the hydrophone from a single, near-surface, transient source at horizontal range r. Such a source could be, for example, a breaking surface wave, which, through the radial oscillations of the associated subsurface bubbles, generates a pulse of sound lasting several seconds.

It is implicit in Eq. (1) that the source strength, Q, is independent of frequency and hence that the source is an impulse occurring at time $t=0$. Such a source has a white spectrum, which is an unrealistic representation of the sound from breaking waves. For our purpose, however, this is of no consequence because the normalization used in forming the noise coherence function is such that the source spectrum cancels out. Indeed, the fact that the coherence function depends on the propagation conditions but is independent of the spectral shape of the sources makes it attractive as the basis of noise inversion techniques for characterizing the ocean environment. (If two or more types of noise process were present, for instance, wind sources and shipping, then the spectral shapes of the sources would influence the coherence function. This, however, is not the situation we are considering.)

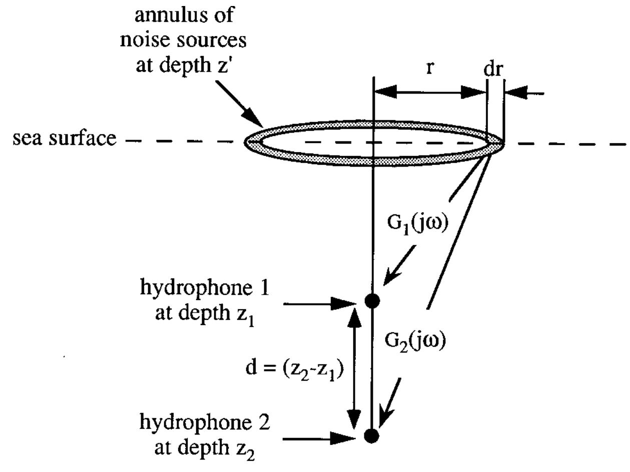

FIG. 1. Schematic showing a vertically separated pair of hydrophones centred on an annulus of near-surface noise sources. The Green's functions G_1 and G_2 are the Fourier transforms (with respect to time) of the velocity potential of a single noise pulse at each of the sensors, and hence are functions of angular frequency, ω.

Obviously, Eq. (1) can be integrated to yield the two familiar spherical spreading terms representing the source and its negative image in the surface. Although Eq. (1) is seemingly more complicated than these two simple terms, there is nevertheless an advantage to be gained by using the inversion integral expression for G when it comes to establishing a complete and exact model of the surface-generated noise field.

II. THE NOISE MODEL

As in previous models of surface-generated noise,[1,13] the sources are assumed to be independent and Poisson distributed in a plane beneath the sea surface. The surface is split into concentric annuli centred on the receiver pair, as shown in Fig. 1, and the contribution to the cross-spectral density of the noise at the receivers from the sources in each annulus is established from Carson's theorem. The cross-spectral density of the noise from the sources within all the annuli is then found by integrating over the whole surface.

This procedure, which is discussed in Ref. 13, leads to the following range integral for the cross-spectral density:

$$\overline{S_{12}}=4\nu\pi\int_0^\infty rG_1G_2^*\,dr, \tag{3}$$

where the overbar and the asterisk denote an ensemble average and complex conjugation, respectively, and ν is the mean rate of source pulses per unit area of surface. The subscripts 1 and 2 identify the Green's functions at receiver depths z_1 and z_2, respectively, from sources within the annulus at range r and of thickness dr. When the wave number formulation in Eq. (1) for the Green's functions is substituted into Eq. (3), the expression for the cross-spectral density takes the form of a triple integral:

$$\overline{S_{12}}=\frac{\nu Q^2}{4\pi}\int_0^\infty\int_0^\infty\int_0^\infty \frac{pp'}{|\eta|^2}F(p,z_1)F^*(p',z_2)$$
$$\times J_0(pr)J_0(p'r)r\ dr\ dp\ dp', \tag{4a}$$

where p' is a dummy horizontal wave number, and

$$F(p,z)=e^{j\eta(z-z')}-e^{j\eta(z+z')}=-2je^{j\eta z}\sin\eta z'. \tag{4b}$$

The integral over range in Eq. (4a) is the Bessel function closure relation,

$$\int_0^\infty rJ_0(pr)J_0(p'r)dr=\frac{\delta(p-p')}{p}, \tag{5}$$

which is a convenient result, since the appearance of the Dirac delta function means that one of the two remaining integrals can be performed immediately, to yield

$$\overline{S_{12}}=\frac{\nu Q^2}{4\pi}\int_0^\infty \frac{p}{|\eta|^2}F(p,z_1)F^*(p,z_2)dp. \tag{6}$$

Equation (6) is an exact expression for the cross-spectral density that is valid provided both sensors lie below the noise sources. It can be further reduced by splitting the integration range into two regions, from zero to k and k to infinity. By making a change of variable, and with a little algebraic manipulation, the cross-spectral density then becomes

$$\overline{S_{12}}=\frac{\nu Q^2}{2\pi}\left\{\int_0^k \frac{e^{-j\eta d}}{\eta}(1-\cos 2\eta z')d\eta\right.$$
$$\left.+\int_0^\infty \frac{e^{-2\xi z_0}}{\xi}(\cosh 2\xi z'-1)d\xi\right\}, \tag{7}$$

where

$$d=z_2-z_1 \quad \text{and} \quad z_0=(z_1+z_2)/2 \tag{8}$$

are the sensor separation and mean sensor depth, respectively. Since the first integral in Eq. (7) depends on sensor separation but not absolute position, it represents the homogeneous component of the noise field. The second integral is a function of the mean depth of the sensors and hence represents the spatially inhomogeneous noise. Notice that the source depth, z', appears explicitly in both integrals.

No approximations have been made in arriving at Eq. (7). If, now, the two terms containing the source depth are approximated to second order by their series expansions, the integrals can be expressed explicitly in terms of elementary functions. Partial integration then leads to the result

$$\overline{S_{12}}\approx\frac{\nu Q^2k^2z'^2}{\pi}\left\{\frac{je^{-jkd}}{kd}+\frac{(e^{-jkd}-1)}{(kd)^2}+\frac{1}{(2kz_0)^2}\right\}; \tag{9}$$

and, to the same order of approximation, the power spectrum at each of the receivers is

$$\overline{S_{ii}}\approx\frac{\nu Q^2k^2z'^2}{2\pi}\left\{1+\frac{1}{2(kz_i)^2}\right\}, \tag{10}$$

where the subscript $i=1$ or 2. Thus to this level of approximation, the coherence between the noise fluctuations at the two sensors is

$$\Gamma_{12}=\frac{\overline{S_{12}}}{\sqrt{\overline{S_{11}}\cdot\overline{S_{22}}}}$$
$$\approx\frac{2\left\{\frac{je^{-jkd}}{kd}+\frac{(e^{-jkd}-1)}{(kd)^2}+\frac{1}{(2kz_0)^2}\right\}}{\left\{1+\frac{1}{2(kz_1)^2}\right\}^{1/2}\left\{1+\frac{1}{2(kz_2)^2}\right\}^{1/2}}. \tag{11}$$

This expression for the coherence function is independent of the source depth, z', which cancels out when the cross-spectrum is normalized by the power spectra.

If the inhomogeneous terms are ignored, Eq. (11) becomes

$$\Gamma_{12}\approx 2\left\{\frac{je^{-jkd}}{kd}+\frac{(e^{-jkd}-1)}{(kd)^2}\right\}, \tag{12}$$

which is Cron and Sherman's result[11,12] for the coherence of deep-water, surface-generated noise. The more complete expression, in Eq. (11), containing the inhomogeneous as well as the homogeneous terms, was derived by Isakovich and Kur'yanov[14] and, using a different approach, by Buckingham,[15] in both cases in connection with low-frequency ambient noise generation in the ocean. It is clear from Eqs. (9) to (11) that the inhomogeneous noise is negligible at sensor depths greater than a wavelength. At shallower depths, however, the inhomogeneous noise component may be a significant or even a dominant contributor to the field. For instance, in the case of infra-sonic noise at a frequency of 1 Hz and a depth of 40 m, the ratio of the inhomogeneous to homogeneous components in the power spectrum [Eq. (10)] is approximately 18:1.

The series approximation leading to Cron and Sherman's expression in Eq. (12) is valid when $2kz'<1$, which is also the condition that identifies a source and its negative image in the surface as a dipole.[16] When this condition is not satisfied, that is, when the source depth is greater than a small fraction of a wavelength, Eq. (12) no longer provides an accurate description of the coherence. The more general expression in Eq. (7), on the other hand, represents the coherence exactly, however deep the source plane is beneath the surface.

III. EXACT TREATMENT OF THE SOURCE DEPTH

To investigate the effect of source depth on the noise coherence, the second-order series approximations for the trigonometric and hyperbolic functions in Eq. (7) must be abandoned. Returning to the exact result for the cross-spectral density of the noise in Eq. (7), the integrands may be expressed in terms of exponentials, yielding

$$\overline{S_{12}}=\frac{\nu Q^2}{4\pi}\left\{\int_0^k[2e^{-j\eta d}-e^{-j\eta(d+2z')}-e^{-j\eta(d-2z')}]\right.$$
$$\left.\times\frac{d\eta}{\eta}+\int_0^\infty[e^{-2\xi(z_0-z')}+e^{-2\xi(z_0+z')}-2e^{-2\xi z_0}]\frac{d\xi}{\xi}\right\}. \tag{13}$$

Each of the terms in this expression is an exponential integral,[17] and each has a lower limit of zero, as a consequence of which it diverges. However, all is not lost, for if the lower limit is replaced by ϵ and the limit taken as ϵ goes to zero, all the divergent terms cancel, leaving an expression for the cross-spectral density that is well-behaved.

To illustrate the cancellation, consider the exponential integral

$$\mathrm{Ei}(-\epsilon)=-\int_{\epsilon}^{\infty}\frac{e^{-x}}{x}\,dx, \tag{14a}$$

which in the limit can be written as

$$\lim_{\epsilon\to 0}\{\mathrm{Ei}(-\epsilon)\}=\gamma+\ln(\epsilon), \tag{14b}$$

where $\gamma=0.577\,215\,7...$ is Euler's constant. Now, on taking the inhomogeneous term in Eq. (13), we find that

$$\begin{aligned}I&=\lim_{\epsilon\to 0}\int_{\epsilon}^{\infty}\{e^{-2\xi(z_0-z')}+e^{-2\xi(z_0+z')}-2e^{-2\xi z_0}\}\,\frac{d\xi}{\xi}\\&=\lim_{\epsilon\to 0}\{2\ln(2\epsilon z_0)-\ln[2\epsilon(z_0+z')]-\ln[2\epsilon(z_0-z')]\}\\&=-\ln\left\{1-\frac{z'^2}{z_0^2}\right\},\end{aligned} \tag{15}$$

which is finite. Following a similar line of reasoning, the exponential integrals of imaginary argument in Eq. (13), representing the homogeneous field, can be expressed as

$$\begin{aligned}J&=\lim_{\epsilon\to 0}\int_{\epsilon}^{k}\{2e^{-j\eta d}-e^{-j\eta(d+2z')}-e^{-j\eta(d-2z')}\}\,\frac{d\eta}{\eta}\\&=\ln\left\{1-\frac{4z'^2}{d^2}\right\}+2\,\mathrm{Ei}[-jkd]-\mathrm{Ei}[-jk(d+2z')]\\&\quad-\mathrm{Ei}[-jk(d-2z')],\end{aligned} \tag{16}$$

a result which is also finite.

At this point, it is convenient to introduce the identity[18]

$$\begin{aligned}\mathrm{Ei}(-jx)&\equiv\int_{\infty}^{x}\frac{e^{-jx}}{x}\,dx\\&=\gamma+\ln(x)-\mathrm{Cin}(x)-j\,\mathrm{Si}(x)+j\pi/2,\end{aligned} \tag{17}$$

where

$$\mathrm{Si}(x)=\int_0^x\frac{\sin x}{x}\,dx \tag{18}$$

is the sine integral and

$$\mathrm{Cin}(x)=\int_0^x\frac{(1-\cos x)}{x}\,dx. \tag{19}$$

With these expressions, in conjunction with Eqs. (15) and (16), the cross-spectral density in Eq. (13) becomes

$$\begin{aligned}\overline{S_{12}}=\frac{\nu Q^2}{4\pi}\Bigg(&\{\mathrm{Cin}[k(d+2z')]+\mathrm{Cin}[k(d-2z')]\\&-2\,\mathrm{Cin}[kd]\}-\ln\left[1-\frac{z'^2}{z_0^2}\right]+j\{\mathrm{Si}[k(d+2z')]\\&+\mathrm{Si}[k(d-2z')]-2\,\mathrm{Si}[kd]\}\Bigg).\end{aligned} \tag{20}$$

Equation (20) is our final formulation of the cross spectrum of the noise. Although it contains two special functions, $\mathrm{Cin}(\cdots)$ and $\mathrm{Si}(\cdots)$, these are entire functions (i.e., they show no singularities) that are particularly fast and accurate to compute using either a series expansion or a rational approximation,[18] depending on the value of the argument. Thus even though $\mathrm{Cin}(\cdots)$ and $\mathrm{Si}(\cdots)$ are defined as integrals, no numerical integration is necessary to evaluate any of the terms of Eq. (20). The power spectrum at either of the receivers is found from Eq. (20) by setting the element spacing, d, equal to zero:

$$\overline{S_{ii}}=\frac{\nu Q^2}{4\pi}\left\{2\,\mathrm{Cin}(2kz')-\ln\left[1-\frac{z'^2}{z_i^2}\right]\right\},\quad i=1,2. \tag{21}$$

Note that, since no approximations are present in Eqs. (20) and (21), they are generally valid for any source depth. By performing a Taylor expansion to second order in the variable $2kz'$, these results reduce identically to the approximate forms in Eqs. (9) and (10). The truncated Taylor series are valid provided $2kz'<1$, which is the condition that identifies a source and its negative image in the surface as a dipole. When this condition holds, Eqs. (9), (10), and the Cron and Sherman expression for the coherence in Eq. (12) are valid. However, for greater source depths or frequencies such that $2kz'>1$, indicating nondipole behavior, the exact expressions in Eqs. (20) and (21) are the appropriate forms to use.

IV. EFFECTS OF SOURCE DEPTH ON COHERENCE

To form the coherence function, Γ_{12}, the cross-spectral density is normalized by the square root of the product of the power spectra, as defined in Eq. (11). Thus from Eqs. (20) and (21), the real and imaginary parts of the coherence function, including the inhomogeneous terms, are

$$\mathrm{Re}(\Gamma_{12})=\frac{\{\mathrm{Cin}[k(d+2z')]+\mathrm{Cin}[k(d-2z')]-2\,\mathrm{Cin}[kd]\}-\ln[1-z'^2/z_0^2]}{\{2\,\mathrm{Cin}(2kz')-\ln[1-z'^2/z_1^2]\}^{1/2}\{2\,\mathrm{Cin}(2kz')-\ln[11-z'^2/z_2^2]\}} \tag{22a}$$

and

$$\mathrm{Im}(\Gamma_{12})=\frac{\{\mathrm{Si}[k(d+2z')]+\mathrm{Si}[k(d-2z')]-2\,\mathrm{Si}[kd]\}}{\{2\,\mathrm{Cin}(2kz')-\ln[1-z'^2/z_1^2]\}^{1/2}\{2\,\mathrm{Cin}(2kz')-\ln[1-z'^2/z_2^2]\}^{1/2}}. \tag{22b}$$

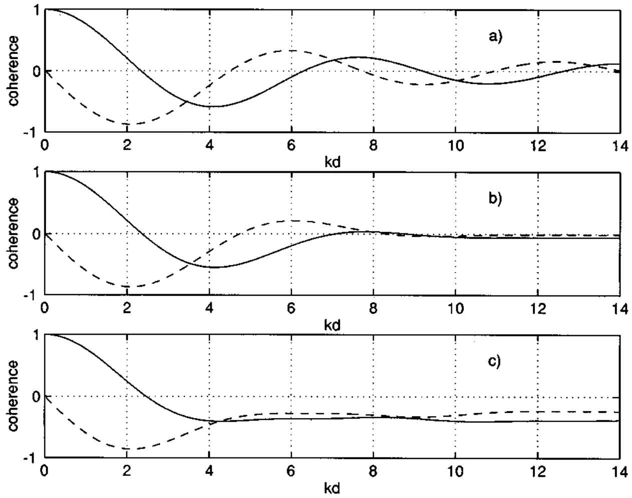

FIG. 2. Effect of increasing source depth on the coherence function. The solid and dashed lines are, respectively, the real and imaginary parts of Γ_{12}, as calculated from Eqs. (23). (a) $z'=0$ (Cron and Sherman); (b) $z'/d=0.25$; and (c) $z'/d=0.5$.

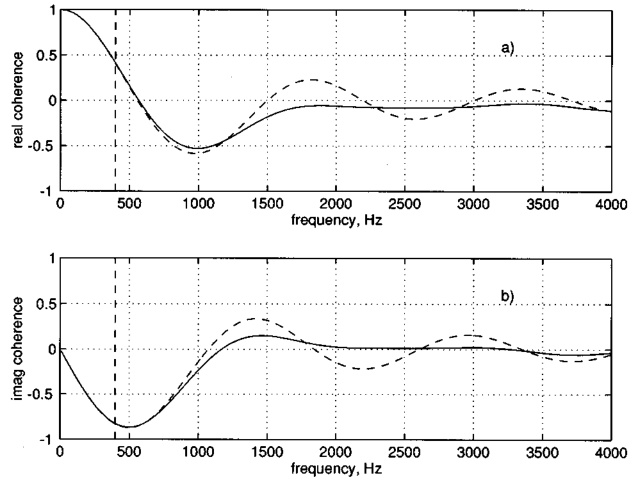

FIG. 3. (a) Real and (b) imaginary coherence curves as a function of frequency for an interelement spacing of $d=1$ m. The dashed curves are from Cron and Sherman's expression, Eq. (12), and the solid lines are from the exact result in Eqs. (23) for a source depth of $z'=0.3$ m. The differences between the curves in both panels occur to the right of the vertical dashed line, indicating the frequency at which $2kz'=1$. To the left of this vertical line, where the source-image pairs act as dipoles, the curves are indistinguishable.

For receiver depths where the inhomogeneous noise, represented by the logarithmic terms, is negligible, these expressions reduce to

$$\mathrm{Re}(\Gamma_{12})=\frac{\{\mathrm{Cin}[k(d+2z')]+\mathrm{Cin}[k(d-2z')]-2\,\mathrm{Cin}[kd]\}}{2\,\mathrm{Cin}(2kz')} \tag{23a}$$

and

$$\mathrm{Im}(\Gamma_{12})=\frac{\{\mathrm{Si}[k(d+2z')]+\mathrm{Si}[k(d-2z')]-2\,\mathrm{Si}[kd]\}}{2\,\mathrm{Cin}(2kz')}. \tag{23b}$$

The coherence in Eqs. (23) is a function of the two-dimensionless variables kd and z'/d. Three examples illustrating the effect of the source depth on the coherence are shown in Fig. 2. In the case considered by Cron and Sherman, where the source depth is infinitesimal and the source-image pairs act as dipoles, the real and imaginary parts of Γ_{12} are oscillatory in character, showing a sequence of zero crossings. As the source depth increases, the curves tend to flatten at the higher frequencies, where the oscillatory behavior is lost, and the higher zero crossings are absent. This behavior can be attributed to the failure, at the higher frequencies, of the dipole description of a source and its image in the sea surface. By way of contrast, at lower frequencies, the curves show the oscillations that are characteristic of surface dipoles, consistent with Cron and Sherman's model, which holds as an accurate representation of the coherence.

Further illustration of the effect of source depth is shown in Fig. 3, where the coherence is plotted as a function of frequency rather than the dimensionless quantity kd. For a source depth of just 0.3 m, it can be seen that the exact expression for the coherence deviates from Cron and Sherman's result above 400 Hz, which is the frequency where $2kz'=1$. Thus at frequencies below 400 Hz, where the surface-dipole description of a source and its image is appropriate, Cron and Sherman's expression for the coherence in Eq. (12) is adequate; but it fails at higher frequencies, where the source depth is comparable to or greater than the wavelength.

Breaking waves inject bubbles beneath the ocean surface. At the instant of formation the bubbles ring for a few milliseconds,[19] thus creating the major part of the wind-generated noise field. The depth of the acoustically active bubbles is uncertain, although an estimate of 1.5 m has been determined from inversions of wave-breaking sound[20] measured at wind speeds of 10 m/s or greater in the sea surface bubble layer.[21] If wave-driven sources do indeed penetrate to depths of order 1 m, then, judging by Figs. 2 and 3, the spatial properties of the ambient noise field could provide a useful measure of the source depth. Although speculative, this argument is supported by evidence that is present in several sets of noise coherence data collected from shallow water sites around the U.K.

Frequency-independent source depths were used to compute the curves in Figs. 2 and 3. It is possible, however, that the resonant bubble sources produced by wave breaking are distributed in depth according to their size, which scales inversely with the resonance frequency. If, as has been suggested by several authors,[22–24] a process of repeated bubble fracture is responsible for the bubble size distribution in the ocean, the bigger bubbles, generating the lower frequencies, may be formed at shallower depths than the smaller, higher-frequency bubbles. As an example of how such a mechanism might affect the coherence of the noise field, suppose that the source depth shows a frequency dependence of the form

$$z'=z_0'(1-e^{-f/f_0}), \tag{24}$$

where f_0 is the e-folding frequency. The function in Eq. (24) is shown in Fig. 4, where it can be seen that at low frequencies the source depth scales approximately linearly with frequency, but with increasing frequency approaches the limit-

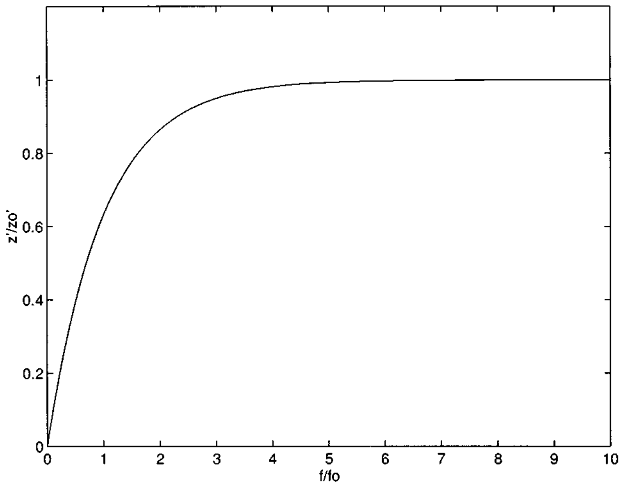

FIG. 4. Frequency dependence of the source depth, as given by the function in Eq. (24).

ing depth, z_0', asymptotically. Figure 5 shows three coherence functions computed from Eqs. (23), using the function in Eq. (24) for the source depth. The differences between the curves in Figs. 2 and 5 are sufficient to suggest that it may be possible to invert noise coherence data to establish the functional dependence of source depth on frequency.

V. CONCLUDING REMARKS

The analysis of deep-water, surface-generated ambient noise presented above illustrates that the depth of the sources is an important factor influencing the vertical coherence of the noise field. When the source depth is significantly less than a wavelength, a source and its negative image in the sea surface act as a dipole, and the real and imaginary parts of the coherence function oscillate about zero as the frequency increases. Such behavior is consistent with that originally predicted by Cron and Sherman[11,12] for the case of an infinitesimal source depth. For deeper sources, however, where the source depth is greater than a wavelength, the real and imaginary parts of the coherence are approximately independent of frequency, showing a form that is quite distinct from that of the Cron and Sherman curves.

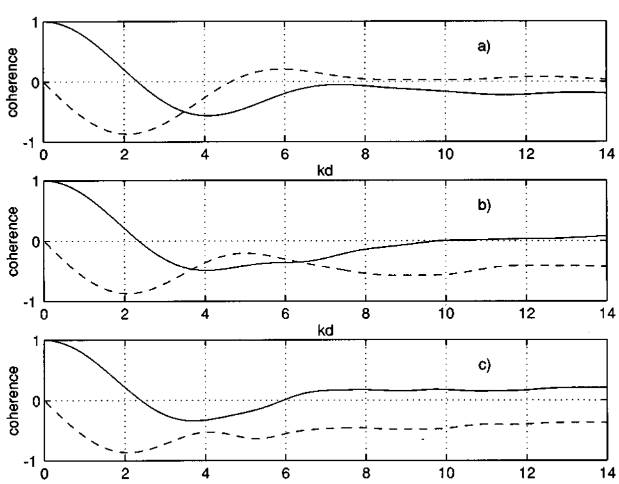

FIG. 5. Real (solid line) and imaginary (dashed line) coherence functions for three different values of the asymptotic source depth, z_0', computed from Eqs. (23) and (24). In all three panels the sensor separation is $d = 1$ m and the e-folding frequency is $f_0 = 2$ kHz. (a) $z_0' = 0.5$ m; (b) $z_0' = 1.0$ m; and (c) $z_0' = 1.5$ m.

Although the source-depth model is idealized, it is useful in providing some insight into the physics of subsurface noise sources and the spatial structure of the associated noise field at depth in the water column. Other factors, such as a rough sea surface, which have been neglected in the model, will also affect the spatial structure of the noise. Of course, in a full numerical treatment of the problem, all such mechanisms should be included.

An obvious feature that is absent from the model is the seabed, which in shallow water channels has a profound effect on the vertical structure of the noise field. The complicated boundary conditions associated with a realistic basement make it difficult, if not impossible, to develop closed-form analytical expressions for the noise coherence in shallow water that include the effect of a finite source depth. The natural alternative is an appropriate numerical technique. In fact, our shallow water noise inversions are based on an algorithm that includes the source depth as an inversion parameter.[25]

The effect of the source depth on the vertical coherence of shallow water noise is qualitatively similar to the behavior exhibited by the deep-water curves. With shallow sources, where the source–image pairs act as dipoles, the real and imaginary parts of the coherence are oscillatory functions of frequency,but both tend to become uniform with increasing frequency. The implication is that, in the higher-frequency regime, the spatial structure of the noise is governed by surface rather than bottom effects. This observation suggests that inversions of the noise coherence aimed at acquiring information about the shallow water environment could provide bottom typing at lower frequencies, where the sources act as dipoles, and source-depth characteristics at higher frequencies. In particular, the higher-frequency noise field may yield the functional relationship between the depth of the wind-driven sources (bubbles) and frequency. Such information is relevant to the use of passive acoustic techniques for inferring the void fraction profile in the sea surface bubble layer and the gas flux across the air–sea interface due to wave breaking.

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APPENDIX: THE GREEN'S FUNCTION

The Green's function, G, of a point source at depth z' beneath a pressure-release surface is the solution of the Helmholtz equation:

$$\nabla^2 G + k^2 G = -Q\,\delta(\mathbf{r}-\mathbf{r}'), \tag{A1}$$

where k is wave number, $\mathbf{r}$ and $\mathbf{r}'$ are the position vectors of the receiver and source, respectively, and Q is the source strength. In cylindrical coordinates, with the axis vertical and passing through the source, this equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right)+\frac{\partial^2 G}{\partial z^2}+k^2G=-Q\frac{\delta(r)}{\pi r}\delta(z-z'), \quad \text{(A2)}$$

where r is horizontal range between source and receiver, and z is depth measured downward from the surface. Symmetry eliminates any azimuthal dependence.

To solve Eq. (A2), subject to the boundary condition

$$G=0 \quad \text{at} \quad z=0, \quad \text{(A3)}$$

we apply a Hankel transform of zero order in range, defined as

$$G_p=\int_0^\infty rG(r)J_0(pr)dr, \quad \text{(A4a)}$$

where p is the horizontal wave number and $J_0(\cdots)$ is the Bessel function of the first kind of order zero. The corresponding inverse transform is

$$G=\int_0^\infty pG_pJ_0(pr)dp, \quad \text{(A4b)}$$

and the Hankel transform of the second derivative with respect to range is

$$\int_0^\infty \frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right)J_0(pr)dr=-p^2G_p. \quad \text{(A4c)}$$

Thus the transformed version of Eq. (A2) is

$$\frac{\partial^2 G_p}{\partial z^2}+(k^2-p^2)G_p=-\frac{Q}{2\pi}\delta(z-z'). \quad \text{(A5)}$$

A Laplace transform over depth, z, is now applied to Eq. (A5), which yields

$$s^2G_{ps}-\dot{G}_p(0)+(k^2-p^2)G_{ps}=-\frac{Q}{2\pi}e^{-sz'}, \quad \text{(A6)}$$

where s is the Laplace transform variable, the integration constant $\dot{G}_p(0)$ is the derivative of G_p normal to the sea surface at $z=0$, and we have used the convention of identifying a transform by subscripting with the transform variable. A second integration constant, $G_p(z=0)$ should also appear on the left of Eq. (A6), but has been set to zero by virtue of the pressure-release boundary condition in Eq. (A3). Equation (A6) provides an algebraic solution for the doubly transformed field G_{ps}:

$$G_{ps}=\frac{-Qe^{-sz'}+2\pi\dot{G}_p(0)}{2\pi(s^2+k^2-p^2)}. \quad \text{(A7)}$$

After applying a standard inverse Laplace transform to Eq. (A7), the unknown constant of integration is found to be

$$\dot{G}_p(0)=\frac{Q}{2\pi}e^{j\eta z'}, \quad \text{(A8)}$$

and the solution for the Hankel transformed field is

$$G_p=-\frac{Q}{2\pi\eta}\{u(z-z')\sin\eta(z-z') -u(z)e^{j\eta z'}\sin\eta z\}, \quad \text{(A9)}$$

where

$$\eta=\sqrt{k^2-p^2} \quad \text{(A10)}$$

is the vertical wave number and $u(..)$ is the unit step (Heaviside) function. On taking the inverse Hankel transform of Eq. (A9), we find that

$$G=-\frac{Q}{4\pi j}\int_0^\infty \frac{p}{\eta}\{e^{j\eta(z-z')}-e^{j\eta(z+z')}\}J_0(pr)dp, \quad z>z', \quad \text{(A11)}$$

which is the solution for the field in Eq. (1) in the text.

An alternative, and somewhat briefer, approach to deriving Eq. (A10) is simply to perform a Hankel transform of the familiar spherical spreading terms representing the source and its image in the surface. The inverse transform for G is then found to be exactly as shown above.

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